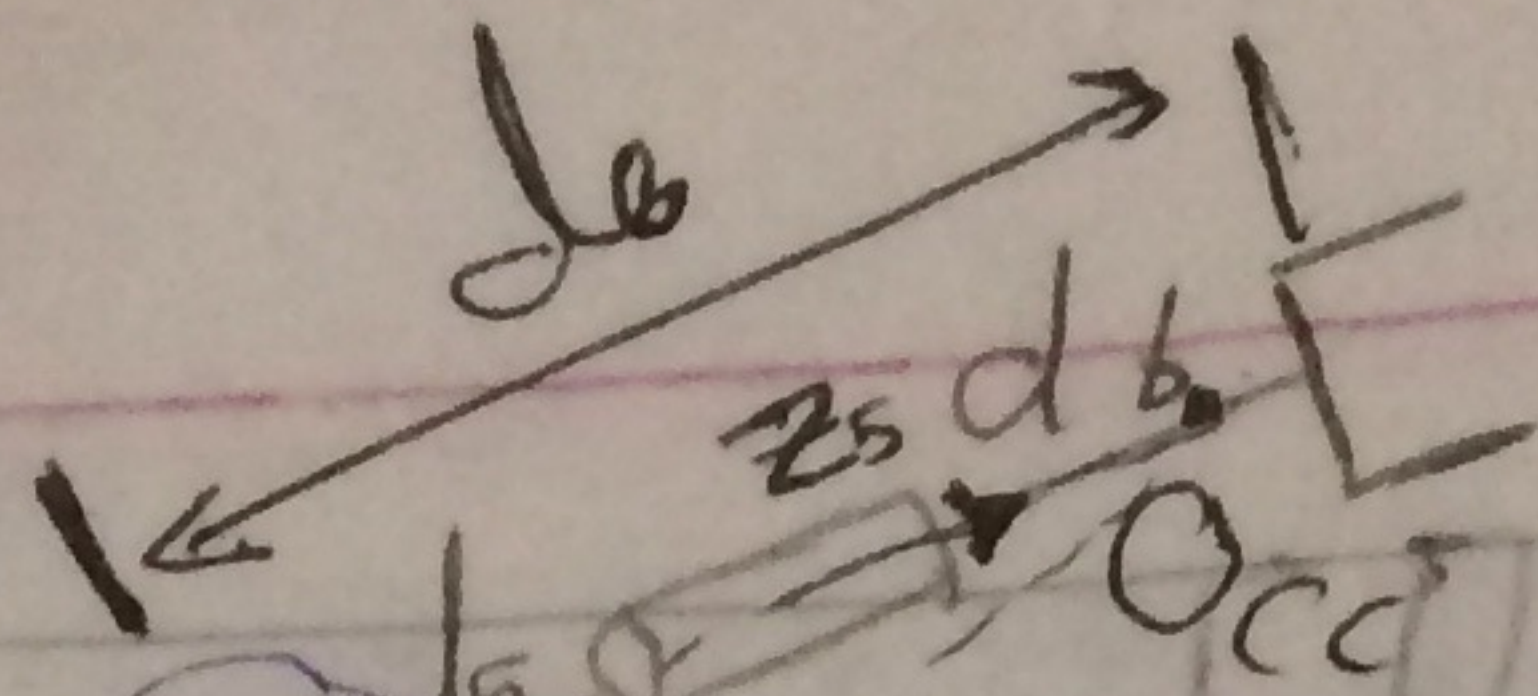


CP 18

see



Robotics  
اولی، دومی، سومی

spherical wrist

3 Angle  
for 3 first joint

وکتورهای  
2 vector

xc

elbow manipulator

yc

closed form = Kinematic Decoupling

Steps →

\* given:  $O_{EE}$  position of end effector, rotation

$$R_{EE} \rightarrow I_{EE}^{base}$$

\* Required:  $q_k$  ex: elbow  $\Rightarrow \theta_1 \rightarrow \theta_6$

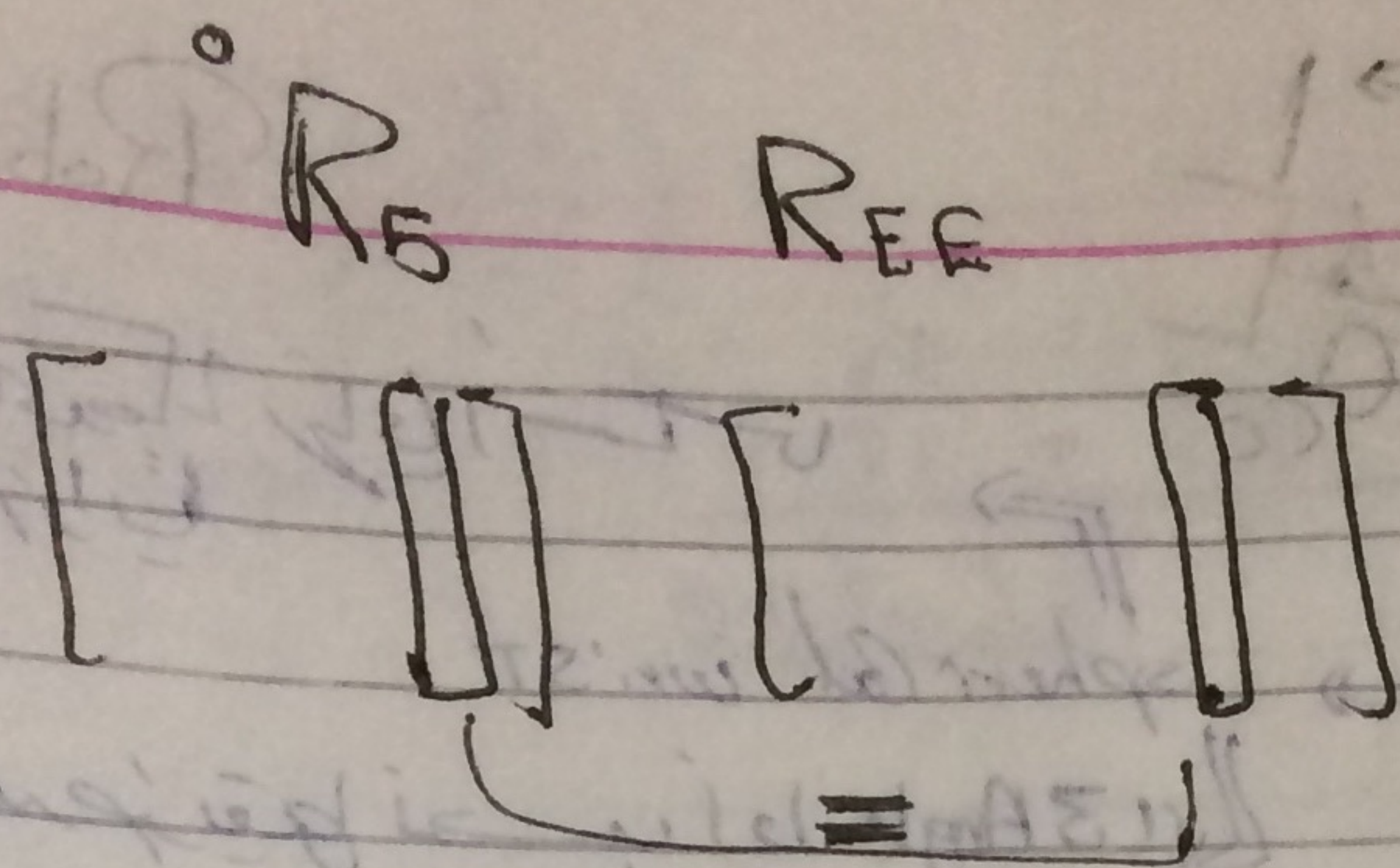
\* Algorithm:

$$1. O_C = O_{EE} \text{ "given"} - d_6 R_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

wrist center.

$$\begin{bmatrix} C30 & -S30 & 0 \\ S30 & C30 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} C20 & -S20 & 0 \\ S20 & C20 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C50 & -S50 & 0 \\ S50 & C50 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





لأن الصيرورة هي

$$O_C = O_{EE} - d_6 R_{EE} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

هذه هي الصيرورة المختلطة.

distance between wrist center and EE

2- From Trigonometric eqn

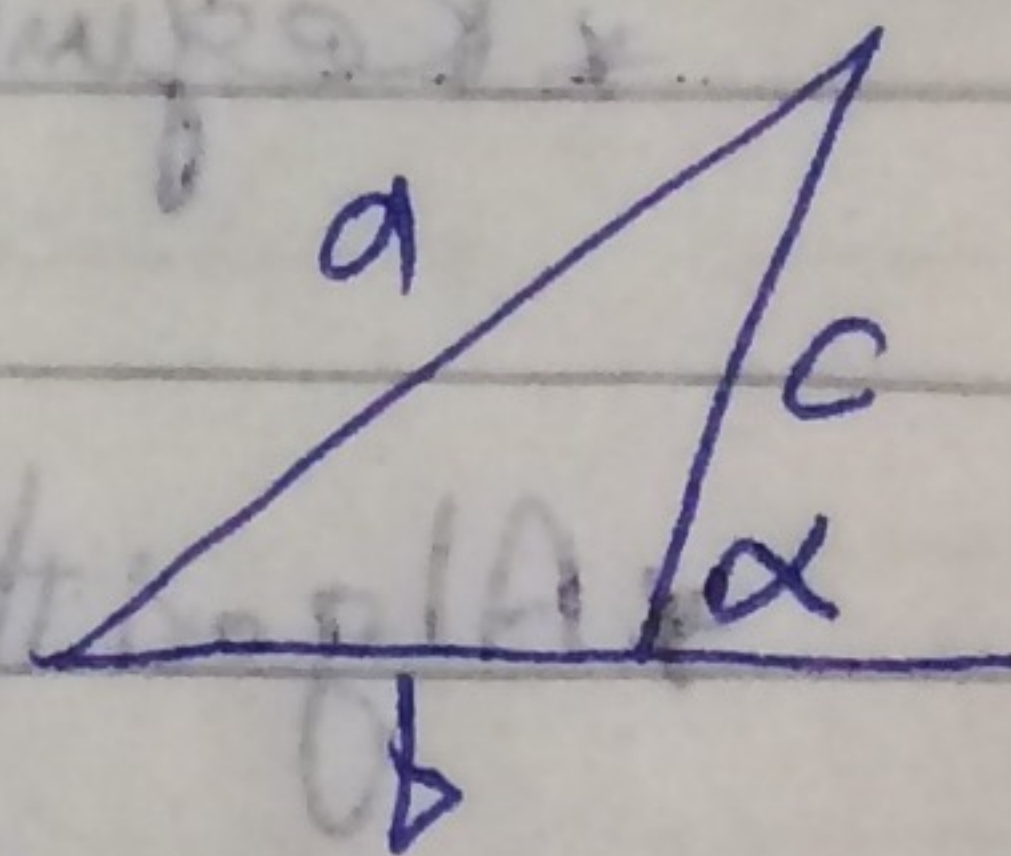
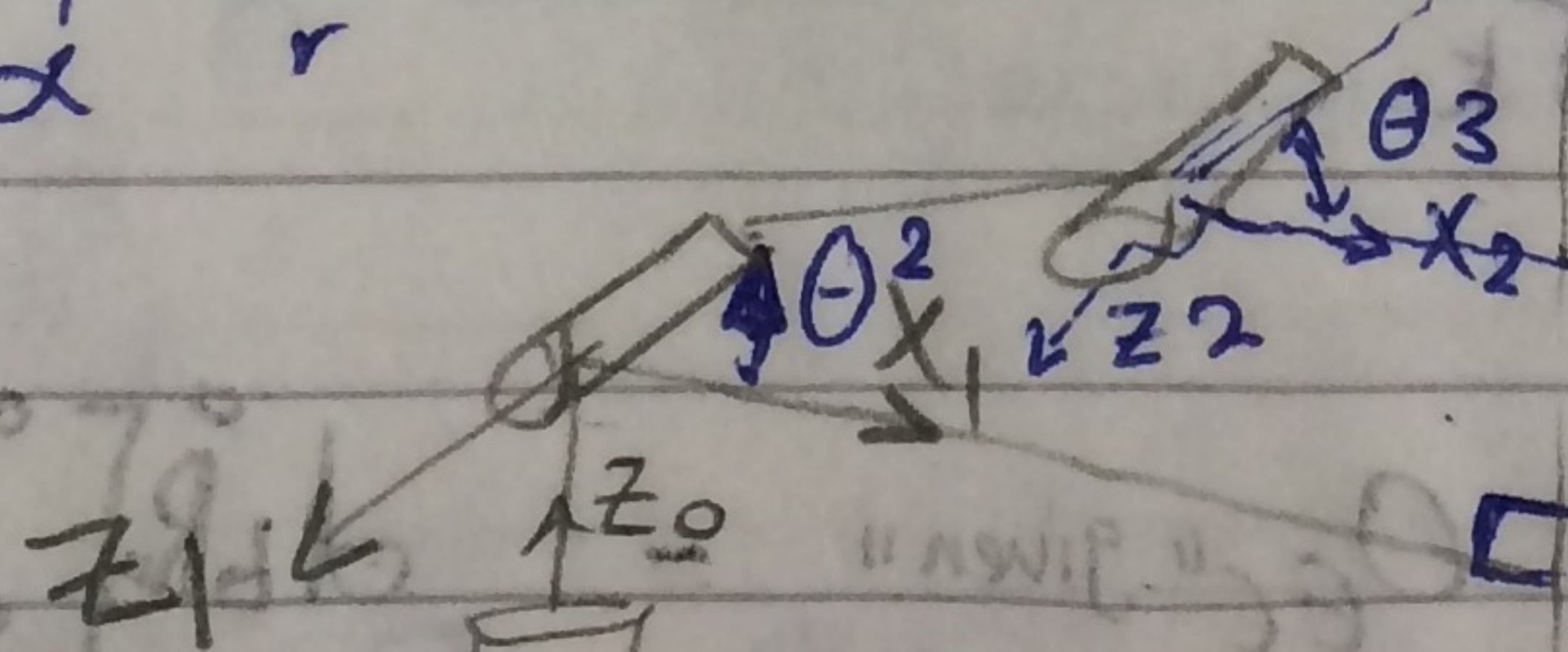
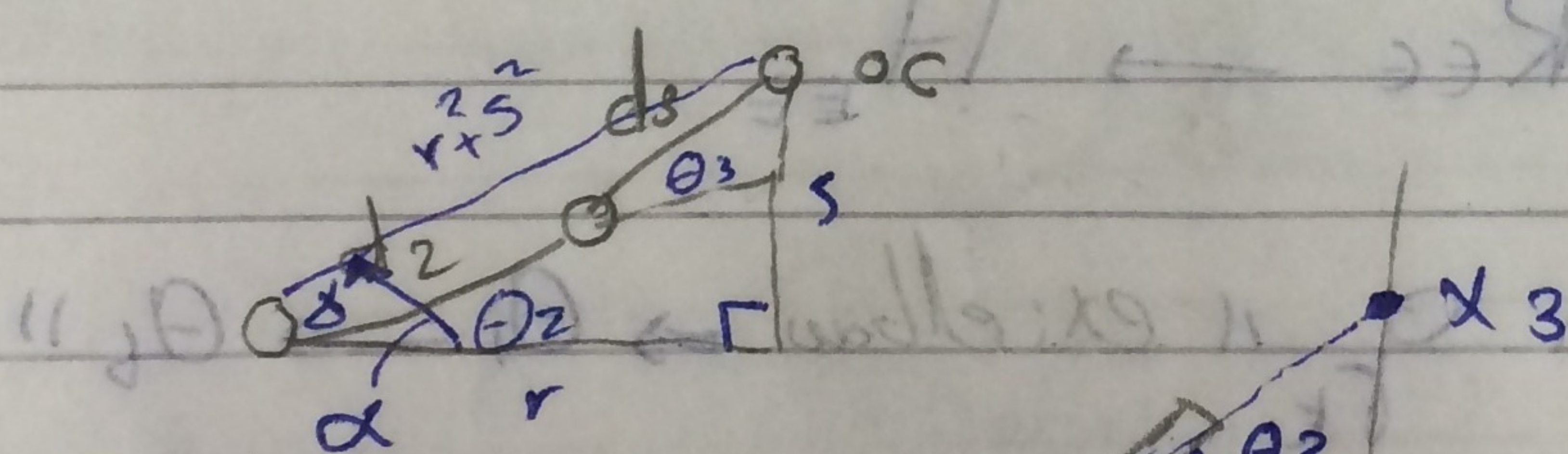
$$\theta_1 = P_1(O_C)$$

$$\theta_2 = P_2(O_C)$$

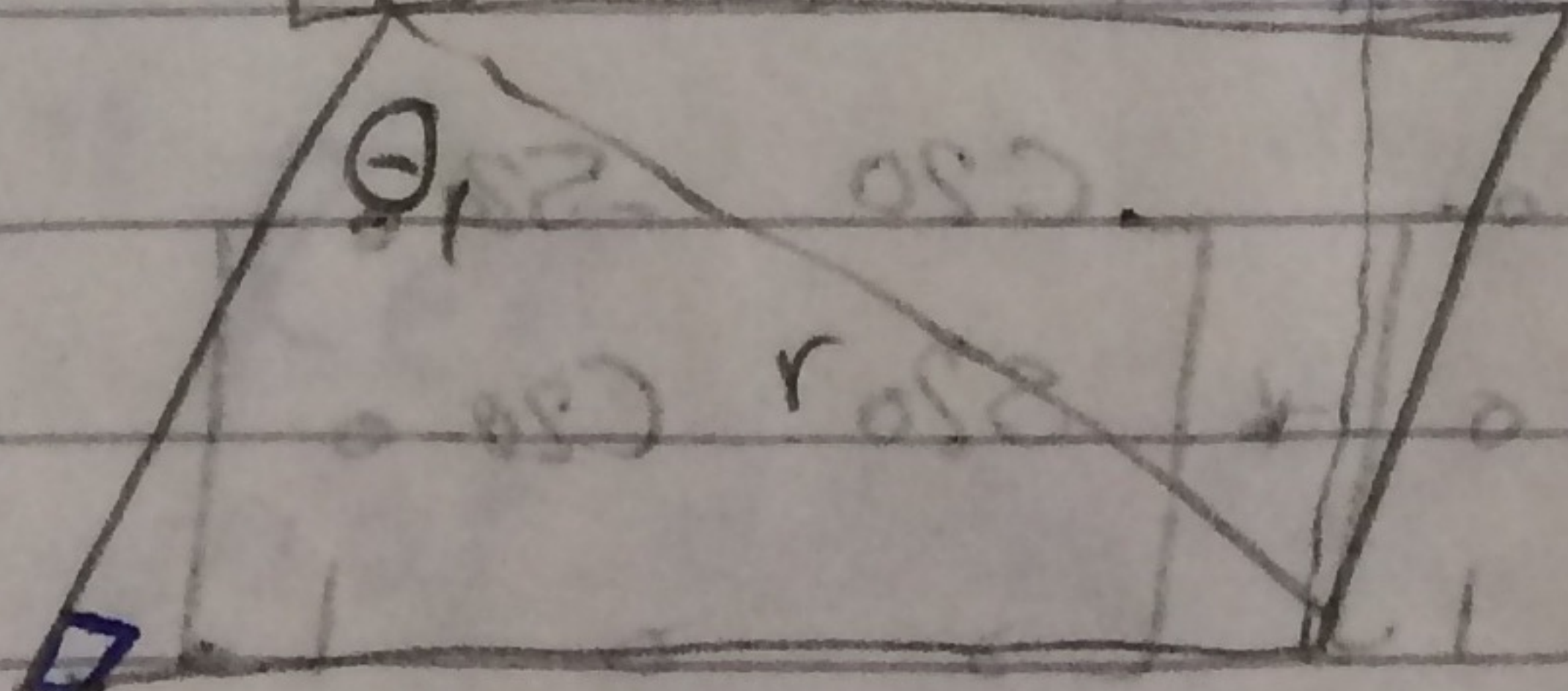
$$\theta_3 = P_3(O_C)$$

على حركته معرفة  $z_c, y_c, x_c$

في النظر من مستوي موازي للـ  $z$  و  $y$  و  $x$



$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$



$x_0$



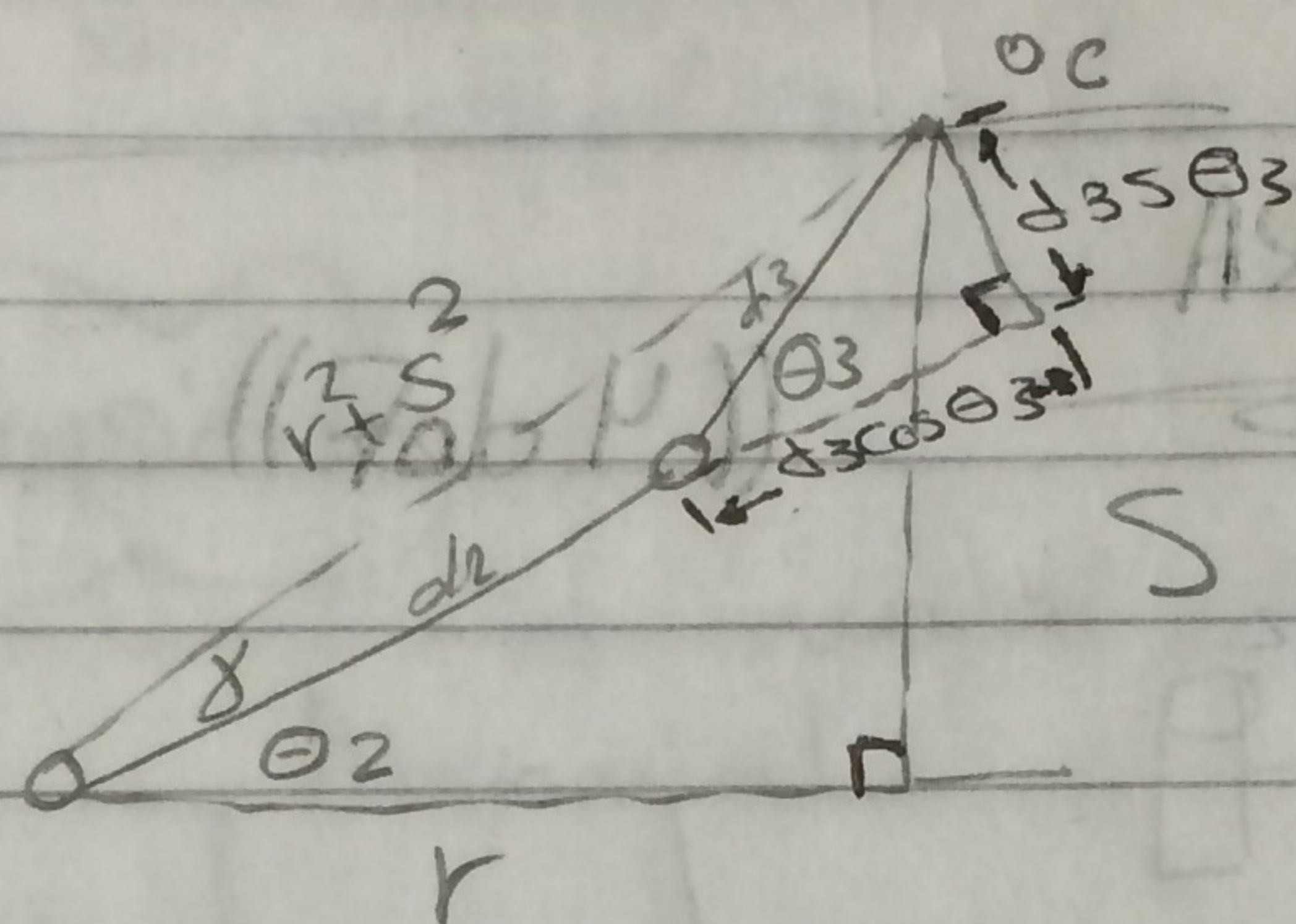
$$\theta_1 = \tan^{-1} \left( \frac{y_c}{x_c} \right)$$

$$\theta_3 = \cos^{-1} \left( \frac{d_2^2 + d_3^2 - (x_c^2 + y_c^2 + (z_c - d_1)^2)}{2 \cdot d_2 d_3} \right)$$

$$\theta_2 =$$

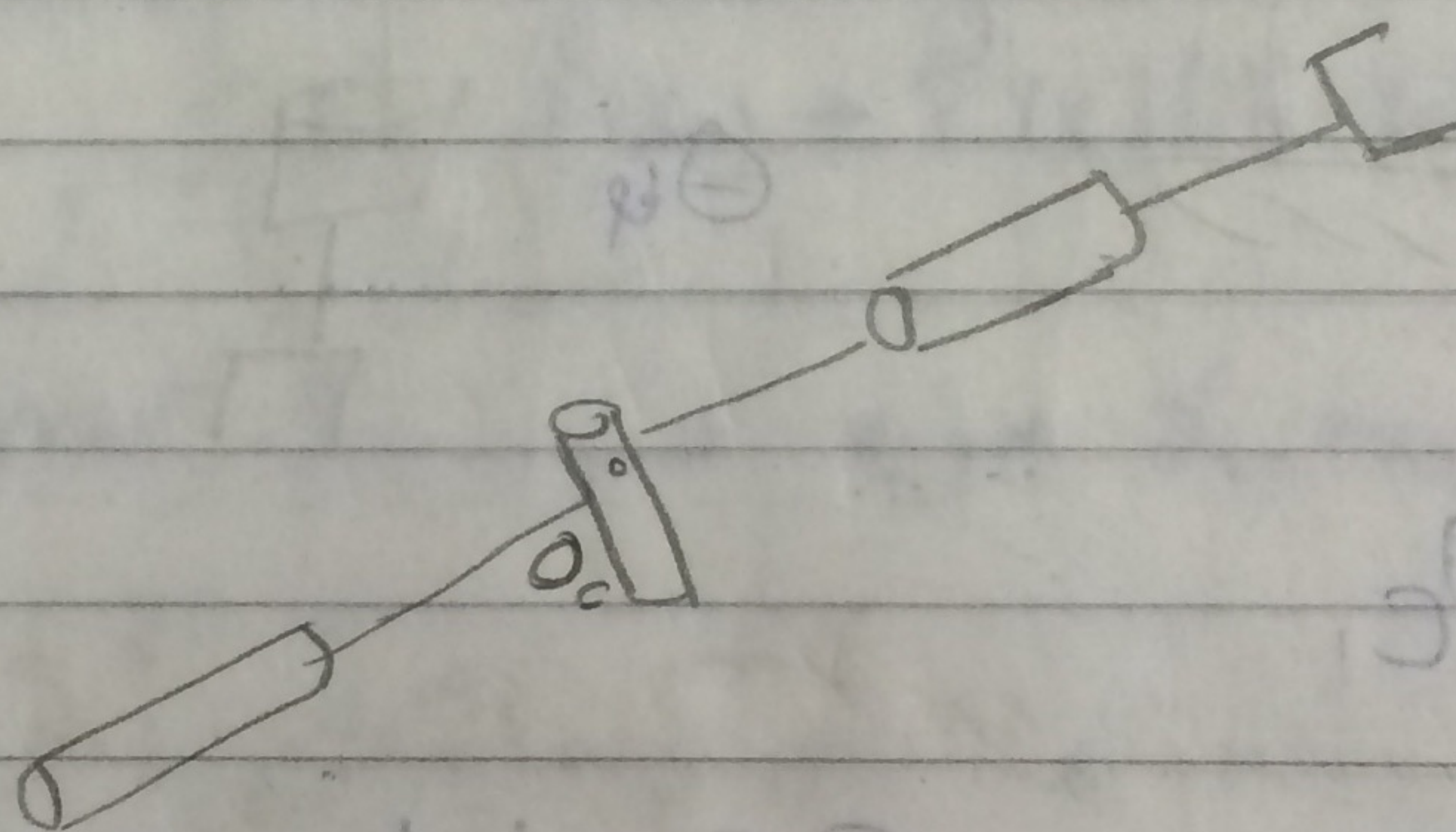
$$\alpha = \tan^{-1} \left( \frac{S}{r} \right)$$

$$\delta = \tan^{-1} \left( \frac{d_3 \sin \theta_3}{d_2 + d_3 \cos \theta_3} \right)$$



3. Orientation.

desired  $\rightarrow \theta_4$   
 $\rightarrow \theta_5$   
 $\rightarrow \theta_6$



$$R_{EE} = {}^0R_3 {}^3R_6$$

${}^0R_3$ : from  $\theta_1, \theta_2, \theta_3$

$${}^3R_6: [{}^0R_3]^{-1} R_{EE}$$

$\rightarrow$  spherical wrist orientation  $\rightarrow$  Euler Angles.

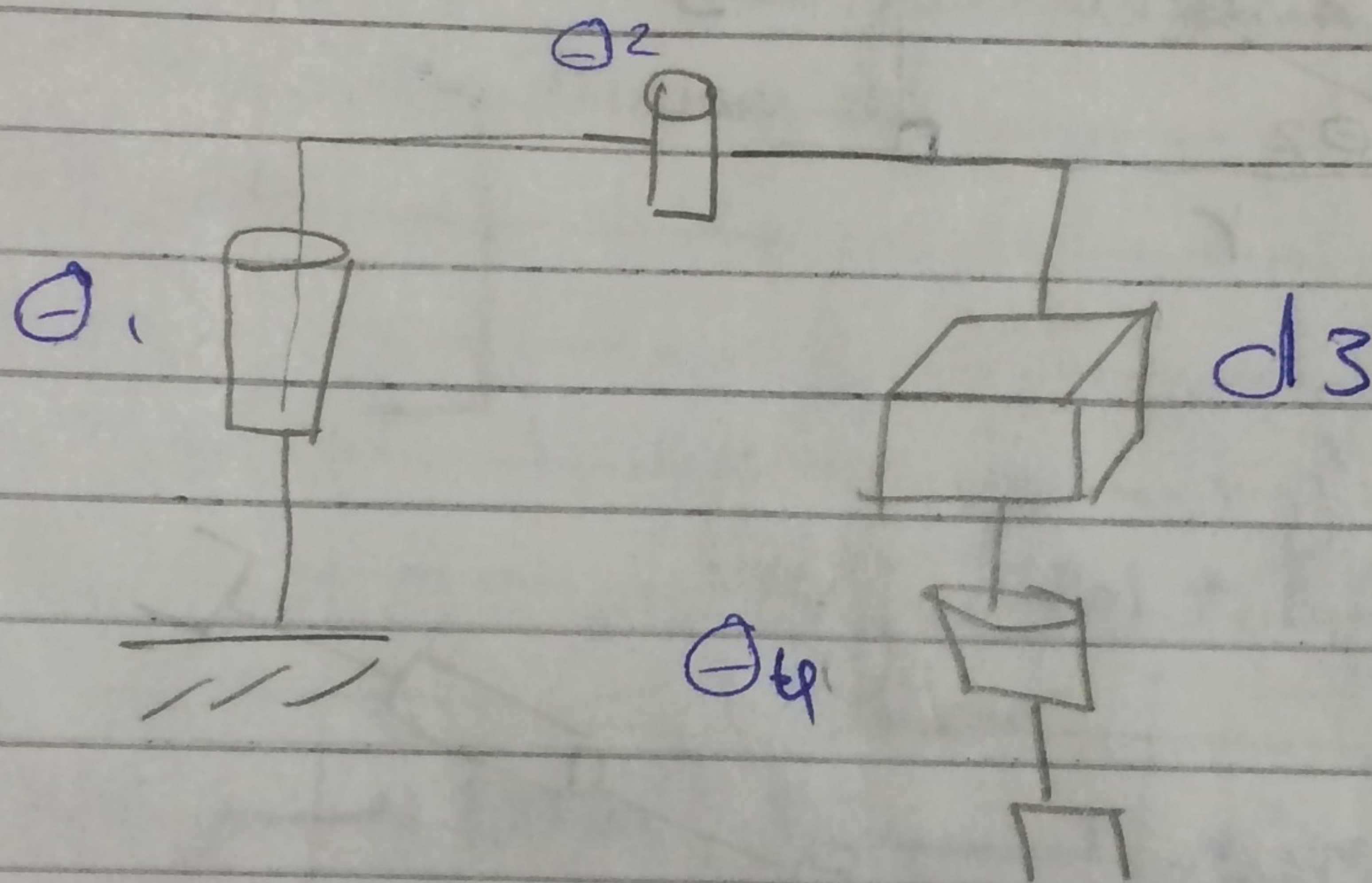


From  ${}^3R_6$  using Euler parametrization:  $\theta_4, \theta_5, \theta_6$

Euler matrix  $\begin{bmatrix} \phantom{\theta_1, \theta_2, \theta_3} \end{bmatrix}$   
 $\theta_1, \phi, \psi$   
 ~~$\theta_4, \theta_5, \theta_6$~~

SCARA

((4 dof))

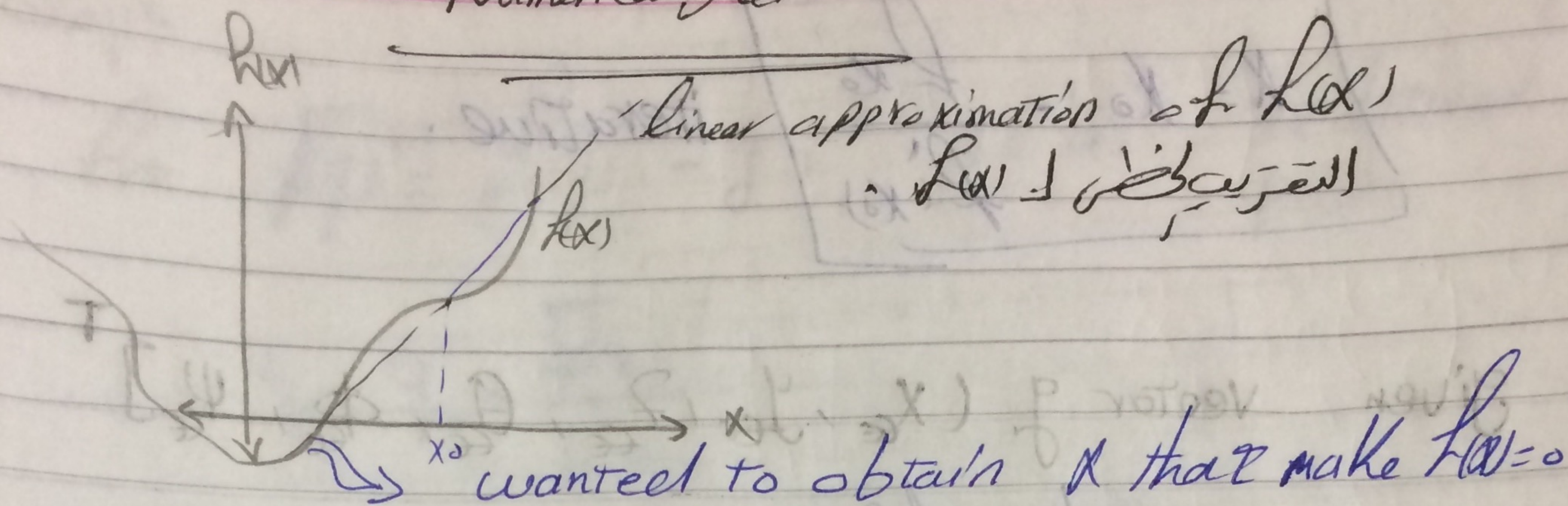


$$\theta_{ec} = \theta_c$$

Position  $\in \mathbb{E}$  independent  $\theta_4$



# Numerical Methods



## Taylor expansion

$f_{ns}$  1 order binomial  
 $f_{ns}$  2nd order  
 $\vdots$

أي دالة تقدر بتغير حيزها و  $as$  no

$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$

linear

كل  $x_0$  range حيزه  $\rightarrow$  error حيزه  $\rightarrow$   $x$

كلما بقدرنا  $x_0$  يكون التكرار أكثر دقة

$$f(x) = 0 \quad \text{linear}$$

$$0 = f(x_0) + f'(x_0)(x - x_0)$$

root

$$-f(x_0) = f'(x_0)(x - x_0)$$

$$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$



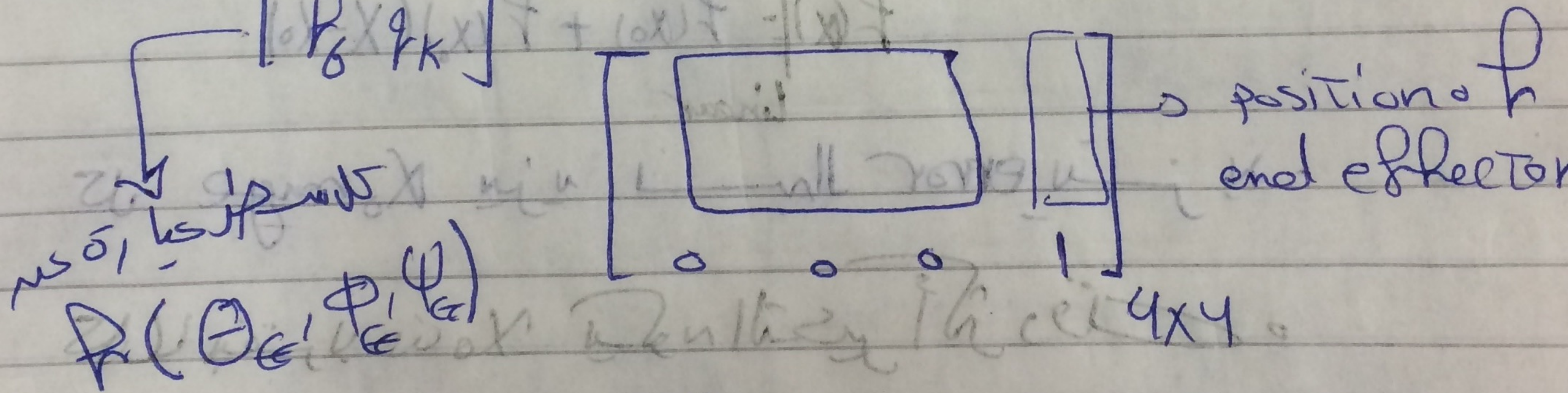
$$X = X_0 - \frac{f(X_0)}{f'(X_0)} \quad \text{iterative.}$$

given vector  $g = [x_e, y_e, z_e, \theta_e, \phi_e, \psi_e]^T$   
desired pose

$N$  (Dof) robot:  $i = 1, \dots, N$

$$h(q_k) = \begin{bmatrix} h_1 q_k \\ h_2 q_k \\ \vdots \\ h_6 q_k \end{bmatrix}$$

from forward kinematics.



6 non linear  $f_{ns} \rightarrow \psi_{ns} = 0$   
 6/p required parameters  $\rightarrow \vec{p}$   
 6/p 6 parameters of end effector  
 $\vec{g}$



Steps

Let  $F(q) = L(q) - \dot{q} = 0$  at the desired pose

$$q = q_0 - \frac{F(q_0)}{F'(q_0)}$$

$$\frac{\partial F_i}{\partial q} = \frac{\partial F_i}{\partial q_1} + \frac{\partial F_i}{\partial q_2} + \dots + \frac{\partial F_i}{\partial q_n}$$

$$\frac{dF_n}{dq} = \text{multi-variable} + \frac{dF_n}{dq_n}$$

$$\frac{\partial F(q)}{\partial q} = \begin{bmatrix} \frac{\partial F_1}{\partial q} \\ \vdots \\ \frac{\partial F_n}{\partial q} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \dots & \frac{\partial F_1}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial q_1} & \dots & \frac{\partial F_n}{\partial q_n} \end{bmatrix} \begin{bmatrix} F_1(q) \\ F_2(q) \\ \vdots \\ F_n(q) \end{bmatrix}$$

Jacobian

اگر من  $F^N$  متغیر واحد

واله واحدی اگر من متغیر

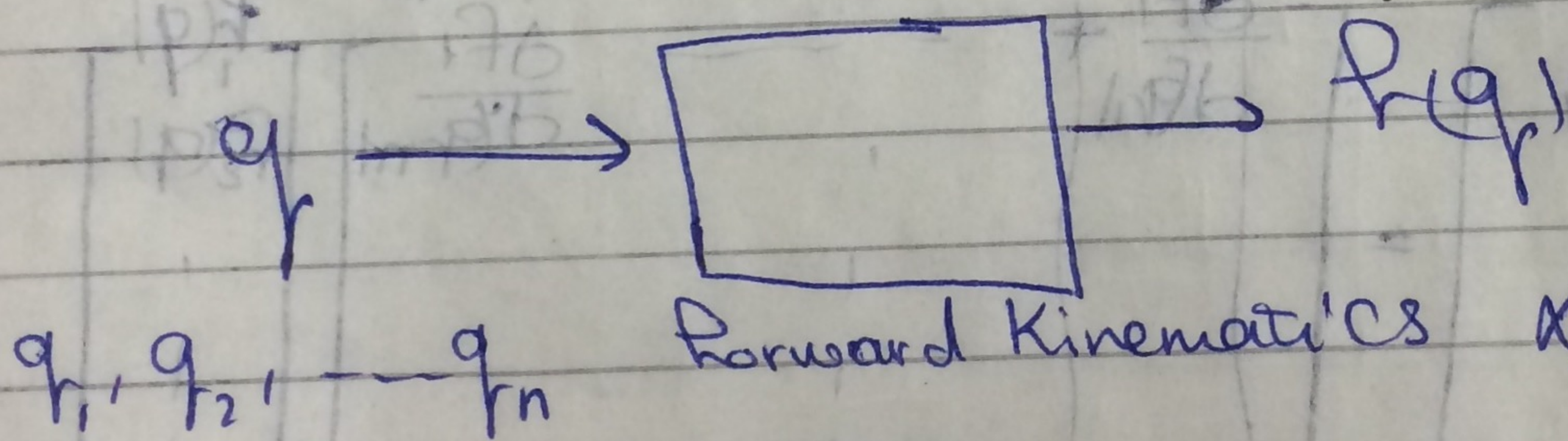


$$q_{n \times 1} = q_{n \times 1} - \underbrace{(F(q))}_{n \times b} \underbrace{F(q)}_{b \times 1}$$

only for square matrix  $\leftarrow$  inverse

pseudo-inverse.

## Artificial intelligence



$x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}, \theta \in \mathbb{R}, \phi \in \mathbb{R}$   
pose

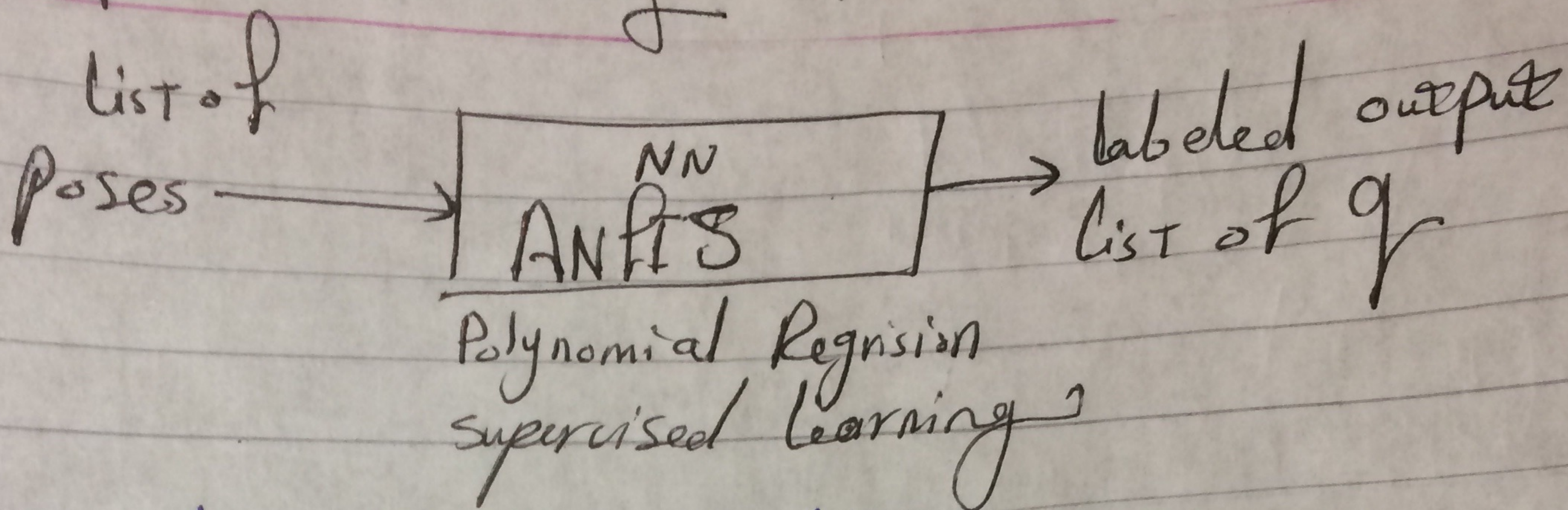
Try ranges for  $q$

1. drive Forward Kinematics.
2. Try ranges of  $q$  and we obtain same Pose

"dataset"  
pose



### 3. Supervised Learning "PSO"



\* we have no direct dataset.

" Trained NN / neural network

